



A Survey on Compressive Sensing by Exploiting the Sparsity of Data in the Transformed Domain for Aggregation

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Abstract: Increasing the lifetime of Wireless Sensor Network remains a prime concern in its design and implementation. The problem is addressed through various techniques for optimizing the data compression techniques and the routing protocols. Conventional compression techniques require combined optimization of data compression and routing protocol to obtain increased efficiency. Compressive Sampling theory allows to decouple the data compression and routing. Hence increased compression ratio itself improves the throughput of the wireless sensor network system. This paper is a survey on Compressed Sensing techniques which uses the alternate sparse sample domain to which the original data can be transformed.

Keywords: Sensor Network, Wavelet, Data Compression, Compressive Sensin.

I. INTRODUCTION

Wireless Sensor Networks(WSN) are densely packed sensor nodes which cooperatively transmit the data acquired by the sensor nodes to a base station. The network formed by the sensors is a self organizing adhoc system. The sensor network is constituted of various small low cost devices with backup power supply. Wireless Sensor Networks are deployed in isolated environments where human interference is less or not possible at all. The geographical area covered by entire network demands multiple hop transmission from sensor to the base station. Hence backup power consumption becomes the prime criterion which determines the lifetime of the WSN. In a WSN, power consumption can be optimized by optimizing both data compression and the routing protocols. Data Compression in the WSN is categorized into Conventional data Compressing method and Compressive Sensing(CS) method. The Conventional data compressing method utilizes the correlation in data during encoding and requires an explicit data communication among sensors. Joint entropy coding approach proposed by Cristescu et al.[6] uses conditional entropy to reduce the number of bits used to encode data. Ciancio et al .[7] and A'ımovı' et al . [8] propose a compressive piece-wise smooth data through distributed wavelet transform. In this method every node transmits its reading to every other node. Each odd node calculates high pass coefficients and each even node calculates low pass coefficients. We can see that in the conventional compression methods the compression techniques need to communicate their data to other nodes for compression. This communication is in addition to the data transmission towards the base node. This clearly shows that the optimization in the conventional method is obtained by joint optimization of the compression technique and the routing protocol. Hence the performance of the compression depends on the joint optimization of the compression technique and the routing protocol which is an NP hard problem. Compressive Sensing on the other hand reduces the global data traffic and distributes energy consumption evenly to lengthen the network life time. Compressive sensing decouples compression and the routing in the sensor network, hence they can be separately optimized. Compressive sensing sampling theory[9][10][11] allows the use of simple encoding process, saves inter data exchange and can deal with abnormal sensor readings. CS data gathering techniques transmit sensor reading jointly rather than separately. In CS data, reconstruction is not sensitive to packet loss. Thus Compressed Sensing data gathering techniques are promising solutions to data aggregation in WSN. This paper is a survey on compressed sensing data gathering techniques. Section II discusses the conventional compressed data gathering with an example. From Section III onwards CS technique which exploits the correlation among the data, is discussed. It transforms the original data set to another domain which sparsely represents the original data. Section IV discusses the Matrix completion method which exploits the low rank matrix recovery theory. Section V discusses the diffusion wavelet technology which allows to find sub network and applies transform on it. Section VI discusses the Matrix Factorization which can be used prior to Matrix completion method.



II. COMPRESSIVE DATA GATHERING

Compressive Data Gathering[1] exploits the correlation between the data in the sensor data set. The correlated data are transmitted to the sink in a combined manner. The data gathering process of CDG is illustrated through an example, Fig.1.1. A small fraction of the routing tree marked in Fig. 1.1(b) is considered for illustration. Leaf nodes initiate the transmission after acquiring the data. Node s₂ generates a random number ϕ_{12} , computes and transmits the value $\phi_{12} d_2$, to s₁. The index i denotes the ith weighted sum ranging from 1 to M. In the same way s₄, s₅ and s₆ transmit $\phi_{14} d_4$, $\phi_{15} d_5$ and $\phi_{16} d_6$ to s₃. Upon receiving the values from all the siblings s₃ computes $\phi_{13} d_3$, and finds the sum of all values in hand ($\phi_{13}d_3 + \phi_{14} d_4 + \phi_{15} d_5 + \phi_{16} d_6$) and transmits it to s₁. Then s₁ computes $\phi_{11}d_1$ and transmits ($\phi_{11} d_1 + \phi_{12}d_2 + \phi_{13}d_3 + \phi_{14} d_4 + \phi_{15} d_5 + \phi_{16} d_6 + \phi_{17}d_7 + \phi_{18}d_8$) the message containing the weighted sum of all readings in a subtree is forwarded to the sink.

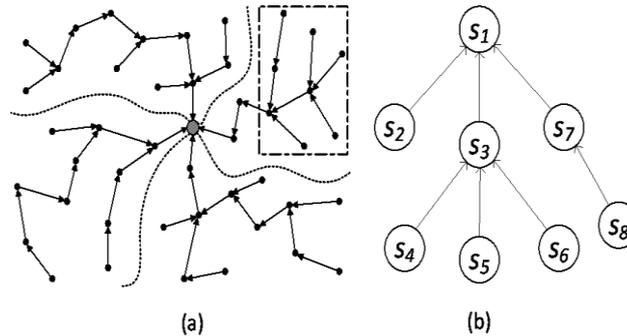


Fig.1.1 (a) A typical routing tree in which the sink has four children. (b) A small fraction of the routing tree marked.

The ith weighted sum can be represented by:

$$y_i = \sum_{j=1}^N \phi_{ij} d_j$$

The sink obtains M weighted sums $\{y_i\}$, $i = 1, 2, \dots, M$ which can be mathematically represented as

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1N} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{M1} & \phi_{M2} & \dots & \phi_{MN} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{pmatrix}$$

This formula can be applied for both scalar and vector quantity of d_i . In the case of d_i being a vector the y_i is also a vector of same size.

For a tree with N nodes which collects M measurements, all nodes send the same number of $O(M)$ messages regardless of their hop distance to the sink. The overall message complexity is $O(NM)$ which is very much less than the $O(N^2)$ the worst case message complexity of the non compressed message transmission. CDG reduces global data traffic besides decoupling the compression and routing, thereby facilitating separate optimisation. Disadvantage of this technique is that, when $M < N$, CDG leaves with solving a set of M linear equations with N unknown variables.

III. COMPRESSIVE SENSING

Actual sensor readings show certain structure because of the spatial or temporal correlations. Hence a transform domain can be identified in which the signal is sparse. Theory of compressive sampling [9][10][111] states that for a sparse discrete signal given by a vector x of size N, reconstruction of x from M random samples are produced by a suitable linear transform ϕ of $x : y = \phi x$ where $M < N$ and the measurement matrix ϕ is of size $M \times N$. Simply stated x can be recovered from the observation y if x is sufficiently sparse subject to some precondition on ϕ . The data set acquired from the sensor set are not usually sparse. But the structure shown by the reading due to spacial or temporal correlation allows to transform the data set sparsely into a different domain $x = \psi s$ for some representation basis ψ of size $N \times N$ and s the coefficient vector in the ψ domain with $\|s\|_0 = K$, where $K \ll N$. Therefore the



measurement vector $y = \Phi \psi$. This measurement vector can be solved by Linear Programming. Also a K -sparse signal can be reconstructed from M measurements with high probability, if M is such that: $M \geq c \cdot \mu^2(\Phi, \Psi) \cdot K \cdot \log N$, where c is a positive constant, Φ is the sampling matrix and $\mu(\Phi, \Psi)$ is the coherence between sampling basis Φ and representation basis Ψ

IV. MATRIX COMPLETION

Given observations Y and the measurement matrix Φ_{MC} , the low-rank matrix completion problem is stated as follows:

$$\min_{X \in \mathbb{R}^{L \times N}} \text{rank}(X) \quad \text{s.t.} \quad Y = \Phi_{MC} \circ X,$$
 A low rank matrix X ($r \ll LN$) of size $L \times N$ can be recovered [2] from randomly selected entries. The observation $Y = \Phi_{MC} \circ X$ where Φ_{MC} is the measurement matrix with $\Phi_{MC}(i,j) = 1$, if (i,j) entry is selected otherwise 0. The operator \circ denotes element wise product i.e., $Y(i,j) = \Phi_{MC}(i,j)X(i,j)$. The defined problem is NP hard, an effective alternative is the nuclear norm relaxation, given by

$$\min_{X \in \mathbb{R}^{L \times N}} \|X\|_* \quad \text{s.t.} \quad Y = \Phi_{MC} \circ X,$$
 where $\| \cdot \|_*$ denotes the nuclear norm, defined as the sum of all singular values. This requires a prior knowledge of the rank, and the observation Y contains $\| \cdot \|_*$ Gaussian noise.

V. DIFFUSION WAVELET

The Classical Wavelet dilates the mother wavelet by the process of 2 to generate a set of wavelet bases. But the diadic dilation generating diffusion wavelet relies on diffusion operator and enables multiscale analysis on general structure such as manifolds or graphs. For an arbitrary Graph G with weighted adjacency matrix $\Omega = [\omega_{ij}]$, the normalized Laplacian Λ characterize the degree of correlation under a certain scale. $\Lambda = [\lambda_{ij}]$ where $\lambda_{ij} = \frac{\omega_{ij}}{\sqrt{d_i d_j}}$ if $i=j$ otherwise

$$\lambda_{ij} = \frac{\omega_{ij}}{\sqrt{\sum_p \omega_{ip} \sum_p \omega_{pj}}}$$

The range space of Λ is partitioned to decompose the signal sampled on a graph in a multiscale manner. A diffusion operator o is constructed from Λ in such a way that the operator and normalized Laplacian Λ share the same eigenvector and all eigen values of the operator o is less than 1. Applying a fixed threshold to eliminate the reducing eigen value and recursively raising o to power of 2, increases the null space and reduces the range space which produces space splitting. With proper sparse basis based on diffusion wavelets high-fidelity recovery for data aggregated from arbitrarily deployed WSNs can be achieved [3]. Also arbitrary network partitions can be developed and temporal correlations can be integrated with the spatial ones, which can significantly reduce energy consumption while maintaining the fidelity of data recovery.

VI. MATRIX FACTORIZATION

The large matrices that appear in modern Machine Learning problems often have complex hierarchical structures that go beyond traditional linear algebra tools, such as Eigen decompositions. Multiresolution analysis, introduces a new notion of matrix factorization that can capture structure in matrices at multiple scales. The resulting Multiresolution Matrix Factorizations (MMFs) not only provide a wavelet basis for sparse approximation, but can also be used for matrix compression and as a prior for matrix completion. Multiresolution matrices are an alternative to the low rank paradigm and in many contexts they better capture the true nature of matrices arising in learning problems. Multiresolution matrix factorization (MMF) [4] uncovers soft hierarchical organization in matrices characteristic of naturally occurring large networks or the covariance structure of large collections of random variables, without enforcing a hard hierarchical clustering. In addition to using MMF as an exploratory tool it can be used for matrix compression, since each intermediate

$$U_\ell \dots U_1 A U_1^T \dots U_\ell^T$$

is effectively a compressed version of Data Matrix A . The wavelet basis associated with MMF is a natural basis for sparse approximation of functions on a domain whose metric structure is given by Data Matrix A .

Multiresolution Analysis (MRA) constructs a sequence of spaces of functions of increasing smoothness by repeatedly splitting each V_j into a smoother part V_{j+1} , and a rougher part W_{j+1} (Figure 5.10). The further we go down this sequence, the longer the length scale over which typical functions in V_j vary, thus, projecting a function to V_j, V_{j+1}, \dots amounts to resolving it at different levels of resolution.

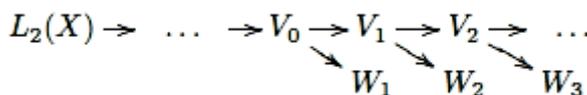


Figure 5.1

Factorization problem focus on how to compresses the matrix A using MRA. The authors found that by extending each ℓ matrix to size $n \times n$ setting,

$$U_{\ell} \leftarrow U_{\ell} \oplus I_{n-\dim(V_{\ell-1})}$$

$U_1 A U_1^T$ the $\phi_1 U \psi_1$ basis becomes

Therefore, similar to the way that Fourier analysis corresponds to Eigen decomposition, multiresolution analysis effectively factorizes A in the form

$$A = U_1^T U_2^T \dots U_L H U_L \dots U_2 U_1$$

where each U_{ℓ} orthogonal matrix must be sufficiently sparse.

VII. CONCLUSION

Conventional compressing method requires joint optimisation of routing and compression to achieve improved throughput. That is routing and compression are coupled. Also solving M equation with N variables is an NP hard problem. Compressive sensing utilizes the correlation in the sensor data acquired, to transform the data into a sparse domain. Here the mother wavelet is dilated and transformed to analyse the structure shown by the correlated data. Matrix completion method exploits the property that a low rank matrix X ($r \ll LN$) of size $L \times N$ can be recovered [4] from randomly selected entries. Here prior knowledge of the rank is required and also the measurement matrix has Gaussian noise. Under multiscale resolution analysis two different methods, the diffusion wavelet and the matrix factorization are discussed. The diadic dilation generating diffusion wavelet relies on diffusion operator and enables multiscale analysis on general structure such as manifolds or graphs. With proper sparse basis based on diffusion wavelets, high-fidelity recovery for data aggregated from arbitrarily deployed WSNs can be achieved [3]. Arbitrary network partitions can be developed and temporal correlations can be integrated with the spatial ones, which can significantly reduce energy consumption while maintaining the fidelity of data recovery. Multiresolution matrices are an alternative to the low rank paradigm and in many contexts they better capture the true nature of matrices arising in learning problems. MMF structure may be used as a “prior” in matrix approximation and completion problems, MMF can be used for matrix compression, The wavelet basis associated with MMF is a natural basis for sparse approximation of functions on a domain whose metric structure is given by Data Matrix.

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